Two Stage No Idle Flow Shop Scheduling To Minimize Rental Cost Including Transportation Time

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***Abstract*— The algorithm of the 2-Stage flow shop scheduling problem under the no idle constraint has been described in this paper, where transportation time is allocated for carrying the jobs from one machine to another. Our research aims to determine the optimal jobs sequence processing with the shortest makespan possible, reducing the machine idle time to zero and lowering the machine rental cost. A heuristic algorithm is made clear with a numerical example.**

***Keywords*— No idle, flow shop, scheduling, transportation time, optimal sequence, total elapsed time, rental cost.**

I. Introduction

Scheduling is assigning resources to regular tasks over a predetermined period to achieve a goal or many goals. In numerous manufacturing and service industries, it often plays a crucial role. In addition, it is vital in circulation settings, transportation, and other overhaul industries.

Flow shop scheduling problems are remarkably complex. Regarding the theory of flow shop scheduling, one of the first discoveries is an algorithm developed by Johnson [1] for 2,3- stage production optimal scheduling problems with a specific goal to minimize the total makespan. J. R. Jackson [2] modified Johnson's [1] method for solving two manufacturing scheduling problems to include scenarios where some jobs only require one stage while others require two stages, and both possible orders might require machines. Ignall, E., & Schrage, L. [3] originated the branch and bound technique to reduce total elapsed time. Further, Yoshida and Hitomi [4] extended the research done by Johnson [1], where setup and processing times were separated in two stage flow shop scheduling problems. Some influential theoretical works on flow shop scheduling problems are provided by Campbell, H.A. et al.[5], Rajendran, C., et al. [6],[7], Lee, Y.H., et al. [8], Wang, X., et al. [9].

When machines are installed in multiple locations where jobs to be processed are planted, these jobs take additional time to transplant from one machine to another due to loading, transferring, and finally unloading time. Maggu and Das [10] defined job transportation time as the sum of all these times. Also, Maggu and Das [11] pioneered the concept of a job block on an equivalent job in two machine flowshop problems involving transportation times of jobs. P. L. Maggu et al.[12] proposed the "job weight and transportation time" of a job in flowshop scheduling problems. Finally, Panwalkar S. S. [13] considered scheduling problems with travel time between machines.

No-idle flow shop scheduling entails no-idle constraints, which means that machines constantly operate with no breaks. Adiri and Pohoryles [14] solved the m-machine no-idle in the flow shop scheduling problem. P. Baptiste and Hguny LK. [15] solved the problem of scheduling n-jobs with three machines under no-idle conditions. Some amendments to Adiri and Pohoryles's [14] work were carried out by Capek et al. [16]. Narain and Bagga [17]worked on n jobs, and two machines flow shop scheduling problems with objective as total flow time under no-idle condition. Goncharov and Sevastyanov [18] have reviewed and approximated the flow shop problem with no-idle constraints. The problem under no idle constraint in which probabilities are associated with processing time is studied by Gupta Deepak and Singh Harminder [19]. Optimum exponential algorithms for the general case and optimal polynomial algorithms for particular scenarios with no idle constraint were added by Nadia Brauner [20].

To date, one of the assumptions in deterministic scheduling theory is the transportation time jobs. In this paper, we relax this assumption since real scheduling difficulties arise when jobs require a certain amount of time to be transferred from one machine to another. Sometimes unavoidable circumstances occur due to the unavailability of funds in an industrialist starting career then, there is no option left, so he has to take the machines on rent to stay financially afloat more easily. In this case, reducing the cost of the rented machinery is our key goal as we aim to create the optimum job sequence possible. In this paper, we have developed a method for two-stage flow shop scheduling problems involving transportation time under no-idle constraints to minimize rental costs.

II. NOTATIONS

i	:	Jobs sequence
J_i	:	Dispensation time of job i on machine J
K_i	:	Dispensation time of job i on machine K
J	:	1st machine
K	:	2nd machine
t_i	:	Transportation time from machine J to the machine K for i^{th} job
$t_{i,2}$	:	The completion time on machine K of i^{th} job
S	:	Optimal sequence using Johnson's technique
$U_1(S)$	:	Utilization time required for machine J in sequence S
$U_2(S)$	:	Utilization time required for machine K in sequence S
C_1	:	Hiring charges of machine J per unit time
C_2	:	Hiring charges of machine K per unit time
L_2	:	Latest time to hire machine K to vanish idle time
R(S)	:	Rental cost for sequence S

III. RENTAL POLICY

According to the current problem's rental policy the standard practice for renting machines is to take them out temporarily and then give them back when they are no longer needed.

IV. PROBLEM FORMULATION

Let us consider that several i (i = 1 to n) jobs ought to be scheduled by machine J and machine K, in the specified order JK. J_i, K_i denotes the dispensation time of job i on machines J and K correspondingly. Let the carrying time required from machine J to machine K for the job i be t_i . Consider the hiring charges of machines J and K to be C_1 and C_2 accordingly. The provided problem is mathematically represented in the matrix form as presented below:-

TABLE I MATHEMATICAL MODEL

Jobs	**Machine J**	**Transportation Time $T_{J\to K}$**	**Machine K**
i	J_i	t_i	K_i
1	J_1	t_1	K_1
2	J_2	t_2	K_2
3	J_3	t_3	K_3
-	-	-	-
n	J_n	t_n	K_n

We aim to get the job processing sequence 'S' to cut machine hiring charges as much as possible.

V. ASSUMPTIONS

- Jobs are independent of each other.
- The breakdown of machines is not taken into account during the scheduling process.
- Transportation from the first to the second machine takes the same time as moving from the second to the first.
- At any given time, just one machine can handle one job.

VI. ALGORITHM

Step-1 Let us call machine J as fictitious machine G and that machine K as a fictitious machine H with dispensation times G_i and H_i for these fictitious machines respectively defined by:

$$G_i = J_i + t_i \tag{1}$$

$$H_i = K_i + t_i \tag{2}$$

Step-2 To determine the optimal sequence for the reduced problem in Step-1, apply Johnson's [1] approach
.

Step-3 Calculate the total elapsed time $t_{n,2}$ by preparing In-Out flow table for the sequence S.

Step-4 Calculate

$$L_2 = t_{n,2} - \sum_{i=1}^{n} K_i \tag{3}$$

Step-5 To vanish idle time, the latest time L_2 is set as starting time to start processing on machine K and prepare in-out table.

Step-6 Lastly, find

$$R(S) = \sum_{i=1}^{n} J_i * C_1 + U_2(S) * C_2 \tag{4}$$

VII. NUMERICAL ILLUSTRATION

Let us consider flow shop scheduling problem having five jobs and two machines, where processing times are represented by J_i, K_i with significant transportation time which is represented as t_i as provided in following Table II . The hiring charges for machines J &K are 4 units and 6 units individually.

TABLE II NUMERICAL PROBLEM

Jobs	**Machine J**	$T_{J\to K}$	**Machine K**
i	J_i	t_i	K_i
1	15	3	8
2	12	2	6
3	5	4	11
4	10	2	5
5	11	5	9

Determine the optimal job processing sequence such that machine hiring costs are kept to a minimum.

A. Solution

Step-1: To begin, we calculate the processing times G_i and H_i as below shown in Table III.

TABLE III PROCESSING TIME INCLUDING TRANSPORTATION TIME

Jobs	G_i	H_i
1	18	11
2	14	8
3	9	15
4	12	7
5	16	14

Step-2: Consequently, using Johnson's [1]approach, we can create the best optimal schedule for the jobs we are operating on. Assume S is the optimum sequence.

$$\mathbf{S = < 3, 5, 1, 2, 4 >} \quad (5)$$

Step-3: In the best-case scenario sequence S, an In-Out flow Table IV is

TABLE IV FLOW IN - OUT TABLE

Jobs	Machine J	t_i	Machine K
3	0-5	4	9-20
5	5-16	5	21-30
1	16-31	3	34-42
2	31-43	2	45-51
4	43-53	2	55-60

Step-4: So, the total elapsed time= $t_{i,2} = 60$

Step-5: Calculate

$$L_2 = 60 - (8+6+11+5+9)$$
$$= 60 - 39$$

$$L_2 = 21 \quad (6)$$

Step-6: The idle time is reduced to zero using IN-OUT table (Table V) with L_2 as beginning time of machine K as shown below:

TABLE V IN–OUT TABLE WITH L_2 AS STARTING TIME FOR MACHINE K

Jobs	Machine J	t_i	Machine K
3	0-5	4	21-32
5	5-16	5	32-41
1	16-31	3	41-49
2	31-43	2	49-55
4	43-53	2	55-60

Step-7:

$$R(S) = \sum_{i=1}^{n} J_i * C_1 + U_2(S) * C_2 \quad (7)$$
$$= 53 * 4 + 39$$
$$= 212 + 234$$
$$= 446 \text{ units}$$

VIII. CONCLUSION

This paper provides a new approach n-job, 2- machine no idle flow shop scheduling problem having transportation time. The proposed algorithm offers a better solution in this paper than Johnson's [1] algorithm for determining an optimal sequence that minimizes the total makespan. As a result, machine rental costs are lowered to a bare minimum, and total elapsed time is reduced as possible.

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