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No Idle Constraint In Flow Shop Scheduling With Transportation Time, Weightage of Jobs And Job Block Criteria

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***Abstract*— In the present paper, a flow shop scheduling model in two stage under no idle constraint has been studied where the transportation time is taken for transferring a job from one machine to another and the two of the jobs has been grouped as a block. Due to their practicality and significance importance in the actual life scenarios, the weight of jobs is also introduced. The study's goal is to introduce a heuristic algorithm that, when implemented, offers an optimal or nearly optimal schedule to diminish the idle time and lowering the rental costs. The effectiveness of the proposed approach is demonstrated through a numerical sample.**

Keywords—scheduling, no idle, job block, weights of jobs, flow shop, transportation time.

I. Introduction

Scheduling is an indispensable process that focuses on the challenges of allocating resources to carry out a series of operations with the objective to identify the optimum solution in light of the need to optimize a function. In the past five decades, there has been considerable attention paid to solve the problem of scheduling. However, Johnson [1] prepared the first triumphant mathematical model that successfully acquired an optimal solution for the two and three stage flow shop scheduling problem. The efficacy of Johnson's model garners significant attention from numerous researchers, who are inclined to explore this avenue. The research conducted by Ignall, E., & Schrage[2], Dannenbring D.G. [3] , J. R. Jackson[4] Yoshida and Hitomi[5], expanded upon their original work by considering a range of parameters and employing different optimality criteria.

From the groundbreaking research conducted by Johnson in 1954, the available scholarly literature pertaining to scheduling models exhibits a notable absence of any discussions regarding the concept of job weightage prior to the year 1980. The weight of a job highlights the degree to which it should be prioritized above other jobs in a scheduling strategy. Miyazaki S.[6] researched flow shop scheduling problems in an effort to reduce the weighted mean flow time of jobs. To improve the weighted mean flow time of jobs, Maggu along with co-authors [7] devised a solution for the n-job, 2-machine flow shop scheduling problem. Chandramouli [8]proposed a heuristic technique aimed at minimizing the overall weighted mean output in a flow shop scheduling problem. Specifically, the problem considered in this research involves a 3- machine, n-job scenario, where each job is associated with weights, transportation times and machine break down intervals. An approach to reduce rental cost for the no idle two-stage flow shop scheduling problem that takes job weighting into account was provided by Gupta, Goel & Kaur [9].

In contemporary industrial practices, it is not uncommon for numerous companies to execute their production processes in separate locations, resulting in the need for transportation time, encompassing loading, moving and unloading, between the distinct machines. Maggu et al. [10] are widely recognized as the pioneers in explicitly considering the transportation factor. These researchers examine a scheduling problem in two-machine flow shop scheduling model, where there are no limitations on the buffer spaces of the machines. In a study conducted by Kise [11] looks into similar issue albeit with the presence of single transporter possessing a capacity of one. The scheduling problems of a flow shop with two machines, one of which is batching machine are investigated by Tang et al.[12]. They also contemplate a transporter for transferring jobs between two machines and a single machine to process jobs that are deteriorating. Further Singla et al. [13] extended the study made by Gupta, Goel & Kaur[9] considering transportation time from processing time under no idle constraint.

The concept of job block holds relevance in practice as it achieves an equilibrium between the expenses incurred by catering to priority clients and those incurred by catering to regular consumers. When specific work orderings are mandated by equipment constraints or by a third-party policy, Maggu, P. L. and Das, G. [14] introduced the fundamental idea of equivalent job per job block in flow shop scheduling. In order to enhance the scope of the study, Anup[15] expanded the research by incorporating probabilities into the analysis of

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job processing time, acknowledging the inherent imprecision in determining the exact duration of job processing. Assigning probabilities with processing time of jobs as the time to process the jobs are always not precise. Gupta, D. et al. [16]studied two stage flow shop scheduling with job block criteria and unavailability of machines using branch and bound technique. Gupta et al. [17] gave an algorithm to minimize rental cost for specially structured two stage flow shop scheduling including transportation time, job block criteria and weightage of jobs. Nature is an ocean of knowledge that motivates living creatures to discover answers to their intricate problems. Additionally, researchers applied this knowledge to solve complex engineering challenges. Several noteworthy references relevant to handle optimization tactics are the works by Malik et al.[18] ,Kumari et al.[19],Singla, Modibbo, Mijinyawa, Malik, Verma& Khurana [20] , Sunita et al.[21][22].

Also, this paper makes an effort to broaden Singla Shakuntala[13] research by incorporating the significant criteria job block. Identifying the most optimal order to complete jobs in order to save down on expensive machine rentals is the focus of the current study.

II. Practical Situation

Numerous practical and empirical scenarios are prevalent in our routine engagement within manufacturing and fabrication environments, wherein diverse tasks necessitate processing on a range of distinct machinery. The practical scenario can be observed in industries such as steel factories, gas plants, and metal refineries, where different grades of steel, gases, and metals are manufactured with varying degrees of importance, specifically in terms of job weightage. The impact of transportation time on manufacturing efficiency becomes significant when the machines used for job processing are located at different geographical locations. For example, transporters that move horizontally on a rail are commonly used to move items around a workshop, such as an electroplating one where metal coatings are applied to various parts. The contractor encountered difficulties in procuring suitable equipment for his construction site, resulting in potential penalties and an inability to meet project requirements for machinery acquisition. So, he exhibits a preference for opting to rent machines rather than purchasing them outright.

A. Assumptions

- Two machines, M_1 and M_2, process the jobs independently of one another in the following order: M_1M_2 with no allowance of any inter-machine transfer.
- There is no way for two machines to process on the same job at the same time.
- Until a job that is being executed can't be finished, the machines' path of action cannot be altered.
- Calculating utilization time does not take machine breakdown or setup times into account.

III. Notations

I	:	Jobs sequence 1,2,…, n
s_1	:	Optimal sequence using Johnson's technique
m_{i1}	:	Processing time of i^{th} job over machine M_1
m_{i2}	:	Processing time of i^{th} job over machine M_2
M_1	:	1^{st} machine
M_2	:	2^{nd} machine
t_i	:	i^{th} Job's transportation time
T_{i2}	:	The completion time of job 'i' on machine M_2
W_i	:	Weightage of job i
$u_1(s_1)$	:	Utilization time required for machine M_1 in sequence s_1
$u_2(s_1)$	:	Utilization time required for machine M_2 in sequence s_1
c_1	:	Hiring charges of machine M_1 per unit time
c_2	:	Hiring charges of machine M_2 per unit time
l_2	:	Latest time to hire machine M_2 to vanish idle time
$r(s_1)$	:	Rental cost for sequence s_1

IV. Problem Formulation

Assume that two machines M_j (j= 1, 2) are to process certain jobs i (1, 2… n). Consider m_{ij} to represent the i^{th} job's processing time on the j^{th} machines M_j. The time it takes to transport a job from machine M_1 to M_2 is denoted by t_i. Let an equivalent job α is defined as (k, m) where k, m are any jobs among the given n jobs such that job k occurs before job m in the order of job block (k, m). Finally, let W_i be the i^{th} job's weightage. The matrix-formatted mathematical representation of the model may be expressed as in TABLE I. Our objective is to identify the sequence of job $\{s_1\}$which helps to keep machines' rental costs down.

TABLE I. MATRIX-FORMATTED MATHEMATICAL FORMULATION

Job	Machine M_1	Transportation Time	Machine M_2	Weight
I	m_{i1}	t_i	m_{i2}	W_i
1	m_{11}	t_1	m_{12}	W_1
2	m_{21}	t_2	m_{22}	W_2
3	m_{31}	t_3	m_{32}	W_3
..	..	..	..	..
n	m_{n1}	t_n	m_{n2}	W_n

A. Theorem

Assume the schedule s = (1,2, 3, …, n) of n jobs are being processed by two machines Y and Z in the order YZ. $\{Y_j'\}_{j=1}^{n}$ and $\{Z_j'\}_{j=1}^{n}$ are the processing times of job j,1 ≤j ≤ n on machine Y and Z respectively. (l,m) is the group job or job block which can be made equivalent to the one job α (called equivalent job α). Job α has processing times Y_α' and Z_α' on the machines Y and Z and are given by:

$$Y_\alpha' = Y_l' + Y_m' - \min(Y_m', Z_l') \quad (1)$$

$$Z_\alpha' = Z_l' + Z_m' - \min(Y_m', Z_l') \quad (2)$$

The proof of the theorem is given by Maggu P.L. and Das G. [14].

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V. ALGORITHM

Step 1: Conceive two hypothetical machines named as X &Y, having the following processing times, X_i' & Y_i' respectively:

$$X_i' = m_{i1} + t_i \quad (3)$$

$$Y_i' = m_{i2} + t_i \quad (4)$$

Step 2: If min (X_i', Y_i') = X_i', then

$$X_i'' = \frac{X_i' - W_i}{W_i} \quad (5)$$

and $Y_i'' = \frac{Y_i'}{W_i}$

If min (X_i', Y_i') = Y_i', then

$$X_i'' = \frac{X_i'}{W_i} \quad (6)$$

And $Y_i'' = \frac{Y_i' + W_i}{W_i}$

Step 3: Consider jobs k and m are working in a job block 'α' with fix order of jobs in which priority is given to job k over m. The concept of a job block can be considered as being equivalent to a single job, denoted as α, where α is defined as (k, m). The processing times will now be determined using equations (1)(2) of job α on invented machines X and Y:

Step 4: Replace jobs k and m with a single job α to transform the given problem into a new one.

Step 5: Get the optimum sequence s_1 while reducing the overall elapsed time by utilizing Johnson's method [1].

Step 6: For schedule s_1, create a flow in- out table and determine total elapsed time.

Step 7: Calculate

$$l_2 = T_{i2} - \sum_{n=1}^{\infty} m_{i2} \quad (7)$$

Step 8: Construct flow in-flow out table for the machines using the most recent time l_2 for machine M_2 to begin processing.

Step 9: Calculate utilization time $u_1(s_1)$ and $u_2(s_1)$ of machines M_1 and M_2 by

$$u_1(s_1) = \sum_{n=1}^{\infty} m_{i1} \quad (8)$$

$$u_2(s_1) = T_{i2} - l_2 \quad (9)$$

Step 10: Finally, calculate

$$r(s_1) = u_1(s_1) * c_1 + u_2(s_1) * c_2 \quad (10)$$

VI. NUMERICAL ILLUSTRATIONS

Consider five jobs and two machines with no-idle flow shop scheduling problems in which processing times, including transportation time and job weightage, are given in TABLE II. Machines M_1 and M_2 have rental costs per unit time of four and five units, respectively. Our goal is to acquire the best possible job sequencing at the lowest feasible amount by considering jobs 2,4 in a block (2,4) that the machines may be rented out for.

TABLE II. DATA SET FOR THE INDICATED PROBLEM

Jobs *I*	Machine M_1 *(m_{i1})*	t_i	Machine M_2 *(m_{i2})*	W_i
1	5	4	4	3
2	8	2	5	4
3	9	5	6	2
4	7	3	12	5
5	10	6	8	1

Solution :

TABLE III. presents, in accordance with Step 1, the two hypothetical machines, X and Y, together with their respective processing times, X_i' and Y_i'.

TABLE III. PROCESS TIME ON HYPOTHETICAL MACHINES

I	X_i'	Y_i'	W_i
1	9	8	3
2	10	7	4
3	14	11	2
4	10	15	5
5	16	14	1

The weighted flow shop times X_i'' & Y_i'' are displayed in TABLE IV. according to Step 2.

TABLE IV. THE WEIGHTED FLOW SHOP TIMES

Jobs *I*	X_i''	Y_i''
1	3	3.66
2	2.5	2.75
3	7	6.5
4	1	3
5	16	15

Select the job block (2,4) and designating it by α, as per **step 3.** Equation (1)(2)is used to calculate how long a single job α will take to process on the two machines:

$$X_\alpha'' = X_2'' + X_4'' - \min(X_4'', Y_2'') = 2.5$$

$$Y_\alpha'' = Y_2'' + Y_4'' - \min(X_4'', Y_2'') = 4.75$$

TABLE V. presents, in accordance with Step-4, the two hypothetical machines, with their processing times X_i'' and Y_i''.

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TABLE V. PORTABLE PROCESS TIMES ON HYPOTHETICAL MACHINES FOR AN EQUIVALENT JOB

I	X_i''	Y_i''
1	3	3.66
α	2.5	4.75
3	7	6.5
5	16	15

As per Step 5; Adopting Johnson's method, the order of the optimum sequence with minimum elapsed time is

$s_1 = \alpha - 1 - 5 - 3.$

$= 2 - 4 - 1 - 5 - 3.$

For schedule s_1, according to **Step 6,** a flow in- flow out TABLE VI. is depicted below:

TABLE VI. FLOW IN-OUT TABLE FOR SCHEDULE S_1

Jobs	M_1	t_i	M_2
2	0-8	2	10-15
4	8-15	3	18-30
1	15-20	4	30-34
5	20-30	6	36-44
3	30-39	5	44-50

Total elapsed time =50

As per **Step-7;** $l_2 = 50 - 35$

$=15$

As per **Step 8,** Create the IN-OUT table as indicated in TABLE VII. to solve the updated scheduling problem.

TABLE VII. FLOW IN-OUT TABLE FOR ROUTE $M_1 \rightarrow M_2$ WITH ZERO IDLE TIME

Jobs I	Machine M_1 IN-OUT	t_i	Machine M_2 IN-OUT	w_i
2	0-8	2	15-20	4
4	8-15	3	20-32	5
1	15-20	4	32-36	3
5	20-30	6	36-44	1
3	30-39	5	44-50	2

As per **Step-9;** $u_1(s_1) = 39$

$u_2(s_1) = 50\text{-}15 = 35$

As per **Step-10;** $r(s_1) = u_1(s_1) * c_1 + u_2(s_1) * c_2$

$= 39 * 4 + 35 * 5 = 331$ units

Hence the above calculated results obtained for machine route $M_1 \rightarrow M_2$ of the optimal sequence $s_1 = \{2, 4, 1, 5, 3\}$ are described in TABLE VIII.

TABLE VIII. COMPARATIVE ANALSIS OF RESULTS

Machine Route $M_1 \rightarrow M_2$	Utilization Time of M_2	Rental Costs
Proposed Algorithm	35 units	331 units
Johnson Algorithm	40 units	356 units

Hence from the above TABLE VIII. , we conclude that the proposed algorithm created for machine route $M_1 \rightarrow M_2$ provides the minimum utilization time and rental cost for optimum solution s_1.

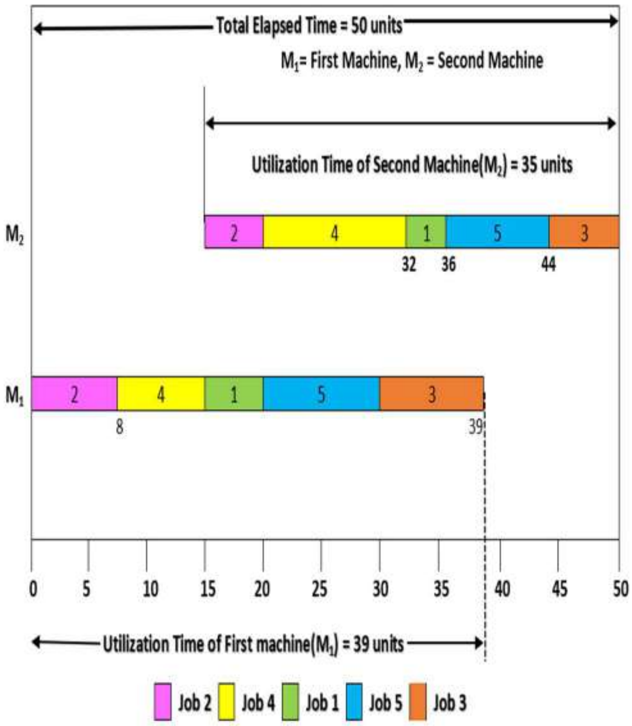

Fig. 1. Gantt chart for the optimum solution $s_1 = \{2,4,1,5,3\}$

The proposed technique is tested by creating a Gantt Chart to see its efficiency in Fig. 1. According to Gantt Chart total elapsed time (make span) is 50 units for optimum solution s_1.

VII. CONCLUSION

The proposed algorithm in this paper provides an efficient solution to no-idle two stage flow shop scheduling problem considering various factors such as processing time, transportation time, job weightage and job block criteria by simultaneously optimizing the rental cost and utilization time. A comparative analysis reveals that the proposed algorithm outperforms the algorithm proposed by Johnson for determining an optimal sequence that minimizes the total makespan. Thus, the method discussed here is practically more applicable and more cost effective. This work can also be extended by considering various parameters like breakdown effect, fuzzy trapezoidal numbers, set up time etc.

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