## SOLIID

## (GEOMMETRY

SUDHIR PUJARA • UPASNA • POONAM SAINI • MINI

A.S. PRAKASHAN, MEERUT A Sister Cancern of Pragati Prakashan. Weerut

# SOLID GEOMETR 

For B.A./B.Sc. I Year (First Semester) Students of M.D.U., Rohtak, K.U., Kurukshetra \& CDLU

SUDHIR PUJARA
Department of Mathematics Govt. College, BAPAULI

POONAM SAINI
Assistant Professor
Department of Mathematics
Guru Nanak Girls College,
YAMUNANAGAR

## MINI

Assistant Professor
Department of Mathematics
S.A. Jain (PG) College,

Ambala City

UPASNA<br>Associate Professor<br>Department of Mathematics<br>Dayanand Mahilla Mahavidalya,<br>KURUKSHETRA

Ambla

A. S. PRAKASHAN

D-106, Saket, Meerut 250 001, U.P.
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## MATHEMATICS

- Algebra

Calculus
Solid Geometry
Number Theory and Trigonometry
ial Equations

- Ordinary Differential Equation
- Vector Calculus
- Advanced Calculus - Advanced Cartial Differential Equations

Sequences and Series

- Sequeial Function and Integral Transforms Programming in ' C ' and Numerical Methods
- Real Analysis
- Group and Rings
- Dynamics
- Real and Complex Analysis

Ph(1-5H2d Linear Algebra

- Numerical Analysis

C Reserved-Solid Geometry
A.S. PRAKASHAN

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## Syllabus

Note : The question paper will consist of five sections. Each of the first four sections (1-1V) will contain two questions (each carrying 7 marks) and the students shall be asked to attempt one question from each section. Section-V will contain six short answer type questions (each carrying 2 marks) without any internal choice covering the entire syllabus and shall be compulsory.

## Section-I:

General equation of second degree. Tracing of conies. Tangent at any point to the conic, chord of contact, pole of line to the conic, director circle of conic. System of conics. Confocal conics. Polar equation of a conic, tangent and normal to the conic.

## Section-II :

Sphere : Plane section of a sphere. Sphere through a given circle. Intersection of two spheres, radical plane of two spheres. Co-oxal system of spheres

Cones : Right circular cone, enveloping cone and reciprocal cone.
Cylinder : Right circular cylinder and enveloping cylinder.

## Section-III :

Central Conicoids : Equation of tangent plane. Director sphere. Normal to the conicoids. Polar plane of a point. Enveloping cone of a coincoid. Enveloping cylinder of a coincoid.

## Section-IV :

Paraboloids: Circular section, Plane sections of conicoids.
Generating lines. Confocal conicoid. Reduction of second degree equations.

## Contents

1. GYNL SECTIONASGENERAL EQUATION OF SECOND DEGREE ..... 1-29
1 Basia Fads
2. Cowa bection ..... 2

* Bonenal Equation of Second Degree ..... 4
* Fo find the length and equations of the areas of a central conic ..... 13
a The Parabola General ..... 24
Exercise ..... 29

2. TRACING OF CONICS ..... 30-40
1 Traciag of a Central Conic ..... 30
2 Traving of a Parabota ..... 32
Exercise ..... 39
3 TANGENT, NORMAL, POLE AND POLAR ..... 41-53
3. Tangent and Normal to a Conic ..... 41
2 Pove and Polar ..... 47
Exercise ..... 53
4. SYSTEM OF CONICS ..... 54-61
5. To show that one and only one conic can be drawn through five given points. No three of which are collinear ..... 54
6. Intersection of two conics ..... 54
Exercise ..... 61
7. CONFOCAL CONICS ..... 62-77
8. Confocal conics ..... 62
9. Exercise ..... 76
a. POLAR EQUATIONS OF A CONIC ..... 78-98
10. Polar coordinates ..... 78
11. Straight line in Polar form ..... 80
12. Circle in the Polar form ..... 82
13. Conic section in Polar form ..... 83
Exercise ..... 97
14. SPHERE
15. Various forms of Sphere ..... 99
16. Sphere through four Points ..... 104
17. Plane section of a sphere ..... 109
18. Any sphere through a given circle ..... 113
19. Intersection of two spheres ..... 113
20. Sphere and a Line ..... 118
21. Tangents to a Sphere ..... 119
22. Diametral and Polar Plane ..... 129
23. Intersection of two Spheres ..... 135
24. Radical Plane ..... 141
Exercise ..... 148
25. CONE
26. Equation of a cone with vertex at the origin 150
27. Equation of a cone with a given vertex and a conic as a base 154
28. Equation of the right circular cone 159
29. Equation of Enveloping Cone 164
30. Cone Represented by the General Equation of Second Degree 166
31. Prove that the D.C.'s of only generator of a cone, represented by
a homogenous equation satisfy the equation of the cone 170
32. Equation of a quadric cone through the axes 171
33. To show that a cone of second degree can be found to pass through
any two sets of rectangular axes through the same origin172
34. Angle between two lines in which a plane through the vertex of a cone cuts a cone 173
35. Condition for a cone to have three mutually Perpendicular
36. Converse of above result is also true
37. Equation of the tangent plane which can be stated as 179
38. Condition of Tangency 185
39. Equation of the Reciprocal cone 185
40. Condition that a cone may have three mutually perpendicular tangent planes 186 Exercise 189

## 9. CYLINDER

$$
\begin{aligned}
& \text { 1. Equation of a cylinder whose axis and guiding curve are given } 191-200 \\
& \text { 2. Equation of the right circular cylinder in }
\end{aligned}
$$

3. Equation of the right circular cylinder in general form ..... 194
4. Equation of the enveloping cylinder ..... 198
5. THE CONICOID ..... 201-244
6. Conicoid ..... 201
7. Equation of tangency Plane ..... 204
8. Condition of tangency ..... 205
9. The equation of the director sphere ..... 212
10. Equation of the Normal ..... 214
11. Number of Normals from a given point ..... 214
12. Cubic curve through the feet of six normals from a point ..... 215
13. Quadric cone through six concurrent normals ..... 216
14. Polar plane of a point ..... 220
15. Reciprocal Property ..... 221
16. Pole of a given plane ..... 222
17. Condition for two planes to be conjugate ..... 222
18. Polar of a line W.R.T. a conicoid ..... 223
19. To find the condition for the two lines to be conjugate lines W.R.T. to a conicoid ..... 224
20. To find the equation of the enveloping cone from the point ..... 226
21. To find the enveloping cylinder of the conicoid ..... 227
22. Diametral plane property ..... 232
23. Plane section with a given centre ..... 232
24. Intersection of a line and a Paraboloid ..... 235
25. To derive the condition of tangency of a line for Paraboloid ..... 238
26. To find the locus of the point of a intersection of three mutually perpendicular tangent planes to a paraboloid ..... 239
27. Normals to an elliptic paraboloid ..... 240
28. Number of normals ..... 241
Exercise ..... 243
29. PLANE SECTION OF CONICOIDS ..... 245-270
30. Contral plane sections of a central conicoid ..... 245
31. Non-central plane section of a central conicoid ..... 252
32. Circular sections ..... 259
33. Plane sections of the paraboloids ..... 264
Exercise ..... 270
34. GENERATING LINES
35. Generating Lines of a hyperbolic of one sheet 279
36. Generating Lines of a hyperbolic paraboloid 279 Exercise 284
37. CONFOCAL CONICOIDS
38. Confocal conicoids ..... 285
${ }^{27} 1_{-284}$
39. Confocals through a given point ..... 286
40. Confocal touching a given plane ..... 287
41. Confocals touching a given line ..... 287
42. Elliptic co-ordinates ..... 288
43. To prove that the parameters of the two confocals ..... 292
44. Locus of Poles of Planes with respect to confocals ..... 293
45. Normals to the three confocals through a point ..... 293
46. The tetrahedro
conicoid 294
47. Axes of Enveloping Cone ..... 295
48. Equation of Enveloping Cone ..... 295
Exercise ..... 299
49. REDUCTION OF SECOND DEGREE EQUATIONS 1. Introduction 300300-322
50. Intersection of a line and a conicoid ..... 300
51. To find the equatir given point 301 of the chord which is bisected at a5. Principal Direction and Principal Planes 302
52. The form of the equation pal Planes 302 ..... 301a centre as origin equation of a quadric surface referred to
53. Reduction of general 308. To show that by rotationation of second degree 304Exercise 322 rotational axes 305
EXAMINATION PAPER
